Student Workbook

Graphs of Logarithmic Functions

Name: ________________________________ Date: _________________ Class: _________

ESSENTIAL QUESTION

How do transformations of $f(x) = \log_b(x)$ move the vertical asymptote and change the domain — and how is this different from what happens when you transform an exponential function?

Lesson Overview

Every logarithmic graph $f(x) = a\cdot\log_b(x-h) + k$ is a transformation of the parent $y = \log_b(x)$. Unlike exponential functions whose asymptote is horizontal, logarithmic functions have a vertical asymptote at $x = h$. Horizontal shifts move this asymptote and change the domain — the key difference to master in this chapter.

$$f(x) = a \cdot \log_b(x - h) + k$$

a	vertical stretch; if $a < 0$, reflects over the x-axis
h	horizontal shift — the asymptote moves to $x = h$
k	vertical shift (does NOT move the asymptote)

Parent Function & Transformation Summary

- Parent $y = \log_b(x)$: domain $(0,\infty)$, range $(-\infty,\infty)$, asymptote $x=0$, x-int $(1,0)$
- $f(x) + k$ / $f(x) - k$: shift up/down k — asymptote stays $x = 0$
- $f(x - h)$ / $f(x + h)$: shift right/left h — asymptote moves to $x = h$ / $x = -h$
- $-f(x)$: reflect over x-axis — asymptote unchanged
- $a\cdot f(x)$: vertical stretch ($|a|>1$) or compress ($0<|a|<1$) — asymptote unchanged
- Range is always $(-\infty, \infty)$ no matter the transformation

Worked Examples

EXAMPLE 1

Graph $f(x) = \log_2(x) + 3$. State the asymptote, domain, range, and x-intercept.

- The "+3" shifts the parent $y = \log_2(x)$ UP 3 units.
- Vertical shift does NOT move the asymptote: $x = 0$.
- Domain: $(0,\infty)$ — unchanged by vertical shift. Range: $(-\infty,\infty)$.
- X-int: $\log_2(x)+3=0 \rightarrow \log_2(x)=-3 \rightarrow x=2^{-3}=1/8$.

Answer: Asymptote $x=0$; Domain $(0,\infty)$; Range $(-\infty,\infty)$; X-int $(1/8, 0)$

EXAMPLE 2

Graph $g(x) = \log_2(x - 3)$. State the asymptote, domain, range, and x-intercept.

- The "(x−3)" shifts the graph RIGHT 3 units.
- Vertical asymptote moves right 3: $x = 3$. Domain: $(3,\infty)$.
- X-int: $\log_2(x-3)=0 \rightarrow x-3=2^0=1 \rightarrow x=4$.

Answer: Asymptote $x=3$; Domain $(3,\infty)$; Range $(-\infty,\infty)$; X-int $(4, 0)$

EXAMPLE 3

Graph $h(x) = -\log_3(x)$. State the asymptote, domain, range, and x-intercept.

- The negative sign reflects the parent over the x-axis.
- Asymptote: $x=0$ (reflection does not move it). Domain: $(0,\infty)$.
- X-int: $-\log_3(x)=0 \rightarrow \log_3(x)=0 \rightarrow x=1$. Now decreasing instead of increasing.

Answer: Asymptote $x=0$; Domain $(0,\infty)$; Range $(-\infty,\infty)$; X-int $(1, 0)$

EXAMPLE 4

Describe all transformations of $f(x) = 2\cdot\log_5(x + 4) - 1$. State the asymptote and domain.

- $a=2$: vertical stretch ×2. $(x+4)$: shift LEFT 4. -1: shift DOWN 1.
- Asymptote: $x=-4$ (shift left 4). Domain: $(-4,\infty)$.
- X-int: $2\cdot\log_5(x+4)-1=0 \rightarrow \log_5(x+4)=1/2 \rightarrow x+4=\sqrt{5} \rightarrow x=\sqrt{5}-4\approx-1.76$.

Answer: Stretch ×2, left 4, down 1; Asymptote $x=-4$; Domain $(-4,\infty)$

EXAMPLE 5

Write the equation of a log function base 3, shifted right 2 and up 5, reflected over the x-axis.

- Start: $y=\log_3(x)$. Shift right 2: $y=\log_3(x-2)$.
- Reflect: $y=-\log_3(x-2)$. Shift up 5: $y=-\log_3(x-2)+5$.
- Asymptote: $x=2$. Domain: $(2,\infty)$.

Answer: $f(x) = -\log_3(x-2) + 5$; Asymptote $x=2$; Domain $(2,\infty)$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 For $f(x) = \log_3(x) - 4$, state the asymptote, domain, range, and x-intercept.

Hint: Vertical shift does not move the asymptote. Find x-intercept by setting $f(x) = 0$.

2 For $g(x) = \log_2(x + 5)$, state the asymptote, domain, range, and x-intercept.

Hint: (x+5) shifts left 5. Asymptote moves to $x = -5$. Domain: $x > -5$.

3 Describe all transformations of $h(x) = -3\cdot\log_2(x - 1) + 2$.

Hint: Identify a, h, k from $f(x) = a\cdot\log_b(x-h)+k$. List each transformation separately.

4 Write the equation of a log function base 2, shifted left 3 and down 1.

Hint: Left shift: replace x with (x+3). Down shift: subtract 1 outside the log.

5 For $f(x) = \log_4(x + 2) - 3$, find the domain and x-intercept.

Hint: Domain: set argument > 0. X-intercept: set $f(x) = 0$ and convert to exponential form.

Key Vocabulary

Vertical Asymptote	A vertical line $x = h$ that the graph approaches but never crosses. Determined by the horizontal shift h.
Domain of a Log Function	For $f(x)=a\cdot\log_b(x-h)+k$, the domain is (h,∞). The argument $(x-h)$ must be strictly positive.
Range of a Log Function	Always $(-\infty,\infty)$ for any logarithmic function, regardless of transformations.
X-intercept	Found by setting $f(x)=0$ and solving. Convert to exponential form after isolating the log.
Increasing vs Decreasing	If $b>1$: $\log_b(x)$ is increasing. If $0<b<1$: $\log_b(x)$ is decreasing.
Vertical Stretch/Compress	Multiplying by $\|a\|>1$ stretches; $0<\|a\|<1$ compresses. Does not change asymptote or domain.
Horizontal Shift	Replacing x with $(x-h)$ shifts right h units ($h>0$) or left $\|h\|$ units ($h<0$). Moves the asymptote to $x=h$.
Reflection	Multiplying by -1 outside reflects over the x-axis; replacing x with $-x$ reflects over the y-axis.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **What is the vertical asymptote of $f(x) = \log_2(x - 5)$?** Multiple Choice

- **A** $x = 0$
- **B** $x = 2$
- **C** $x = 5$
- **D** $x = -5$

2 **What is the domain of $g(x) = \log_3(x + 4)$?** Multiple Choice

- **A** $(0, \infty)$
- **B** $(4, \infty)$
- **C** $(-4, \infty)$
- **D** $(-\infty, \infty)$

3 **What is the range of $h(x) = -2 \cdot \log_5(x - 1) + 7$?** Multiple Choice

- **A** $(7, \infty)$
- **B** $(-\infty, 7)$
- **C** $(1, \infty)$
- **D** $(-\infty, \infty)$

4 **Find the x-intercept of $f(x) = \log_2(x - 1)$.** Multiple Choice

- **A** $(0, 0)$
- **B** $(1, 0)$
- **C** $(2, 0)$
- **D** $(3, 0)$

5 **Which function has vertical asymptote $x = -3$?** Multiple Choice

- **A** $f(x) = \log(x) - 3$
- **B** $f(x) = \log(x + 3)$
- **C** $f(x) = \log(x - 3)$
- **D** $f(x) = 3 \cdot \log(x)$

Common Mistakes to Avoid

✗ Saying the asymptote of $f(x) = \log(x) + 5$ is $x = 5$ (confusing vertical shift with horizontal shift).

✓ Adding outside the log shifts vertically — the asymptote stays at $x = 0$. Only $(x - h)$ inside moves it to $x = h$.

✗ Writing the domain of $f(x) = \log_2(x - 3)$ as $(0, \infty)$ instead of $(3, \infty)$.

✓ The domain is set by the argument: $(x - 3) > 0 \rightarrow x > 3$. Domain: $(3, \infty)$.

✗ Saying the range of $f(x) = -\log(x) + 7$ is $(-\infty, 7)$ because of the reflection and shift.

✓ The range of every log function is always $(-\infty, \infty)$. Transformations never restrict the range.

✗ Thinking $f(x) = \log(x + 3)$ shifts the graph RIGHT 3.

✓ $f(x+3)$ shifts LEFT 3. To shift right, use $f(x-3)$. The sign inside is counterintuitive.

Math Tips

★ The vertical asymptote of $f(x) = a \cdot \log_b(x-h)+k$ is always $x = h$. Set the argument equal to zero and solve.

★ Domain rule: set the argument > 0 and solve for x. That solution set is the domain interval.

★ Range is always $(-\infty, \infty)$ for any log function — no exceptions. You never need to calculate it.

★ To find the x-intercept: set $f(x) = 0$, isolate the log, then convert to exponential form.

★ Sketch checklist: (1) find asymptote $x=h$, (2) plot x-intercept, (3) plot $(h+b, 1)$, (4) draw the curve approaching the asymptote.